# Illumination patterns for multiple right-conic monochromatic light sources on plane surfaces

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## Abstract

We have proposed a simplified model for multiple-LED based illumination systems. We consider multiple monochromatic point sources, each of which emit a right-conic beam. We attempt to evaluate the system by finding the illumination patterns, their relative positions in respect to each and their relative conic angle to get optimal illumination at the surface. The work is aimed at developing integrated illumination systems for low cost hand held optical spectroscopic devices having image acquisition and processing applications.

## 1. Introduction

There are several instances of applications, where illumination of surfaces are required from a given source(s) [1]. The illumination thus obtained is often then used to acquire images or visual data [2], based on which various measurements and analysis can be made.

For such purposes, it is common practice to have multiple point sources of a given wavelength for illumination. These point sources are often placed on the body of a hemisphere, with the region of interest located at the base of the resultant hemispheres.

In this paper we discuss how the illumination pattern varies for 1 and 2 sources, when placed symmetrically on such a hemisphere, with the base of the hemisphere acting as the region of interest.

## 2. Models and Methods

### 2.1. Single Point Illumination

In this model we have a sphere of radius ***r***. A point source of light is attached to the sphere and is rotated for all possible values of $\boldsymbol{\theta}$ representing the position of a LED on the sphere. The point source makes an angle cone angle of $\boldsymbol{\omega}$. We assume in this model that the center of the light cone, bisects the center of the base of the hemisphere at all values of ***r***, $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$. (Eg. Fig 1)

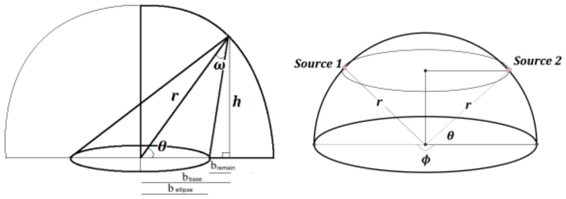
$$h = r \times sin(\theta) \quad \dots (1)$$

$$b_{base} = r \times cos(\theta) \quad \dots (2)$$

$$b_{remain} = \frac{r \times sin(\theta)}{tan(\theta + \omega)} \quad \dots (3)$$

$$b_{ellipse} = b_{base} - b_{remain} = r \times cos(\theta) - \frac{r \times sin(\theta)}{tan(\theta + \omega)} \quad \dots (4)$$

$$a_{ellipse} = r \times tan(\omega) \quad \dots (5)$$




**A** **B**

FIG. 1: Point source of light on surface of a hemisphere
A. Single B. Double

### 2.2 Double Point Illumination

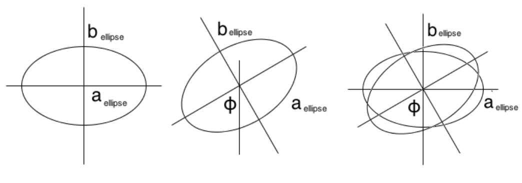
Like the above model, we have two point sources with angles $\boldsymbol{\theta}$ and varying angle $\boldsymbol{\phi}$ between them (Fig 1B). This is mathematically equivalent to two ellipses of semi-major axes $\boldsymbol{a_{ellipse}}$ and $\boldsymbol{b_{ellipse}}$ being rotated about their center. The rotation value is computed from $\boldsymbol{\phi}$ the angle between two point sources. Like the last instance, we assume in this model that the center of the light cone bisects the center of the base of the hemisphere at all values of ***r***, $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ and $\boldsymbol{\phi}$.

The system is described by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots (6)$$

$$\frac{(xcos\phi + ysin\phi)^2}{a^2} + \frac{-(ysin\phi - ycos\phi)^2}{b^2} = 1 \quad \dots (7)$$

We then find the points of intersection between the ellipses, and the resultant area using Gauss-Green [3].




**A** **B**

FIG. 2: Double point source of light on a plane surface
A. Non-overlapped B. Overlapped.

## 3. Results and Discussions

### 3.1 Single Point Illumination

The illumination area varies as a function of $\boldsymbol{\theta}$.

The ratio of the area of illumination is independent of the ***r***. However illumination area increases with $\boldsymbol{r^2}$ with increase in distance ***r***.

This is evident if one looks at the ratio,

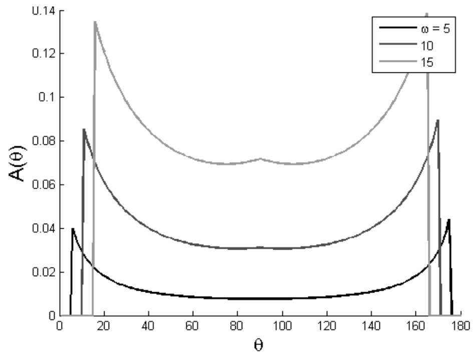
$$A_{ratio} = \frac{Areaofellipse}{Areaofbaseofhemisphere}$$
$$= \frac{r \times (cos(\theta) - \frac{sin(\theta)}{tan(\theta+\omega)}) \times r \times (tan(\omega))}{r^2}$$

… (8)

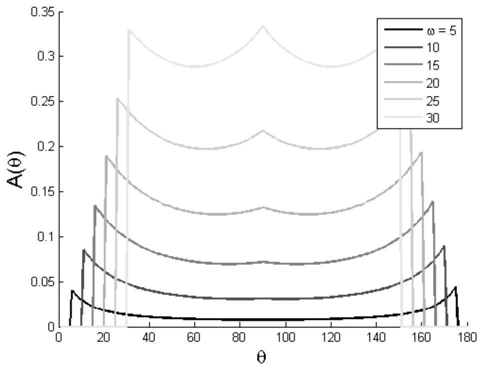
At lower values of $\omega$ (~ $\omega < 15$) the maximum area of illumination (Fig. 3A) is for **lim** $_{\theta \to 0}$ and **lim** $_{\theta \to 180}$.

At moderate values of $\omega$ (~ $\omega < 30$) the maximum area of illumination (Fig. 3A) is for **lim** $_{\theta \to 0}$, **lim** $_{\theta \to 90}$ and **lim** $_{\theta \to 180}$.

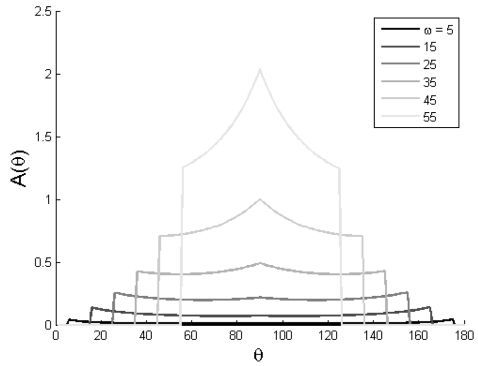
For higher values of $\omega$ (~ $\omega > 30$) the maximum area of illumination (Fig. 3C) is for **lim** $_{\theta \to 90}$.




**A**




**B**



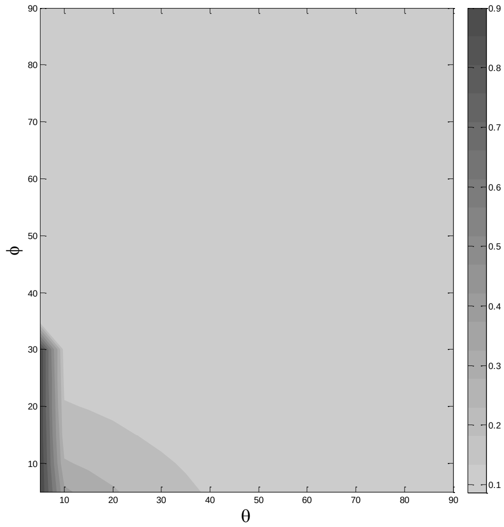

**C**

FIG. 3: Single point illumination.
A. $\omega$ < 15 B. $\omega$ < 30 C. $\omega$ > 30

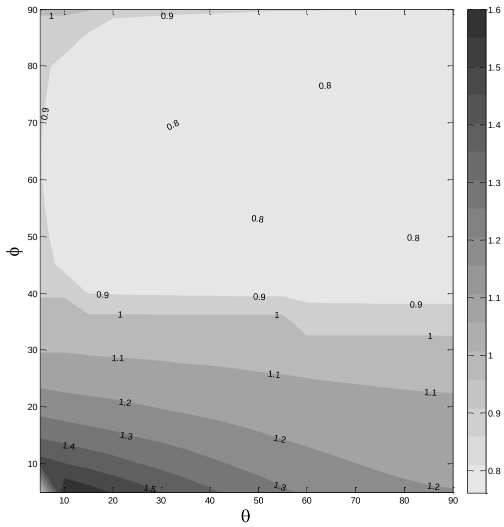
## 3.2 Double Point Illumination

Here (in Fig 4.) we see the variation of $A_{ratio}$ as we vary $\theta$ and $\phi$ while keeping $\omega$ at ***10*** and ***30 degrees.***

We see the significant variation in illumination area across the variant range.




**A**




**B**

FIG. 4: Double point illumination.
A. At $\omega$ = 10 B. At $\omega$ = 30

## Future Work

A more general solution for less granular variation of $\omega$ and a model for multiple (>2) sources can be developed.

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## References


[1] Situ, W., Han, Y., Li, H., and Luo, Y. Combined feedback method for designing a free-form optical system with complicated illumination patterns for an extended LED source. *Optics express* **19**(105), A1022-A1030, 2011.

[2] Shimizu, M., Yano, T., Setomoto, T., Matsui, N., and Tamura. LED illumination apparatus and card-type LED illumination source, *U.S. Patent No. 7,375,381*, 2008.

[3] Hughes, G. B. and Chraibi, M. Calculating Ellipse Overlap Areas, http://arxiv.org/abs/1106.3787 2011.